Classification

Fundamentals of Artificial Intelligence

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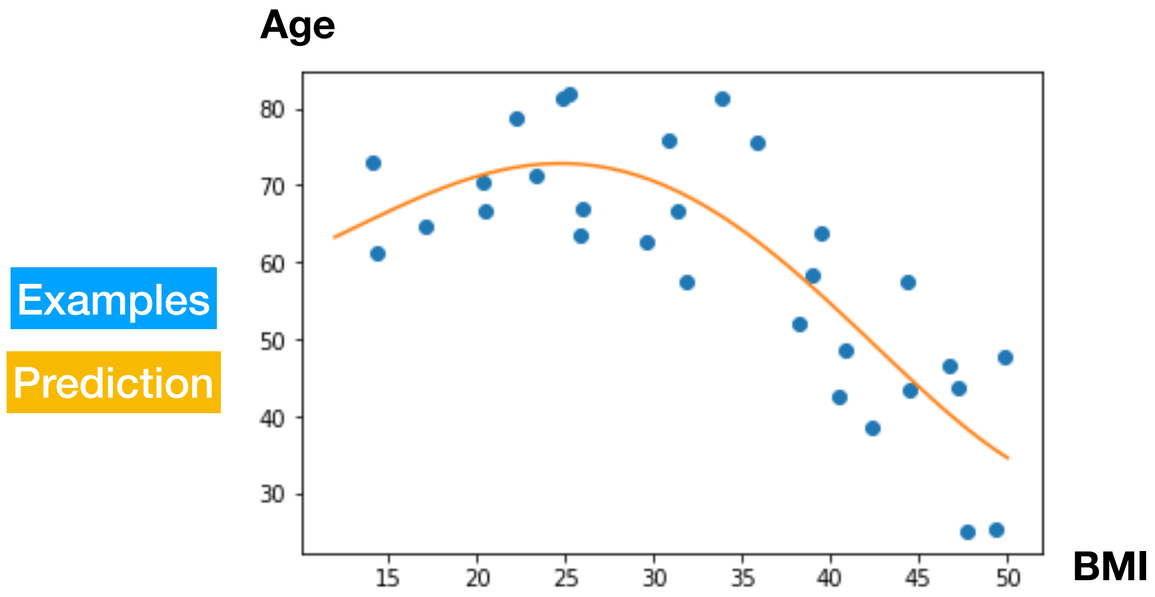
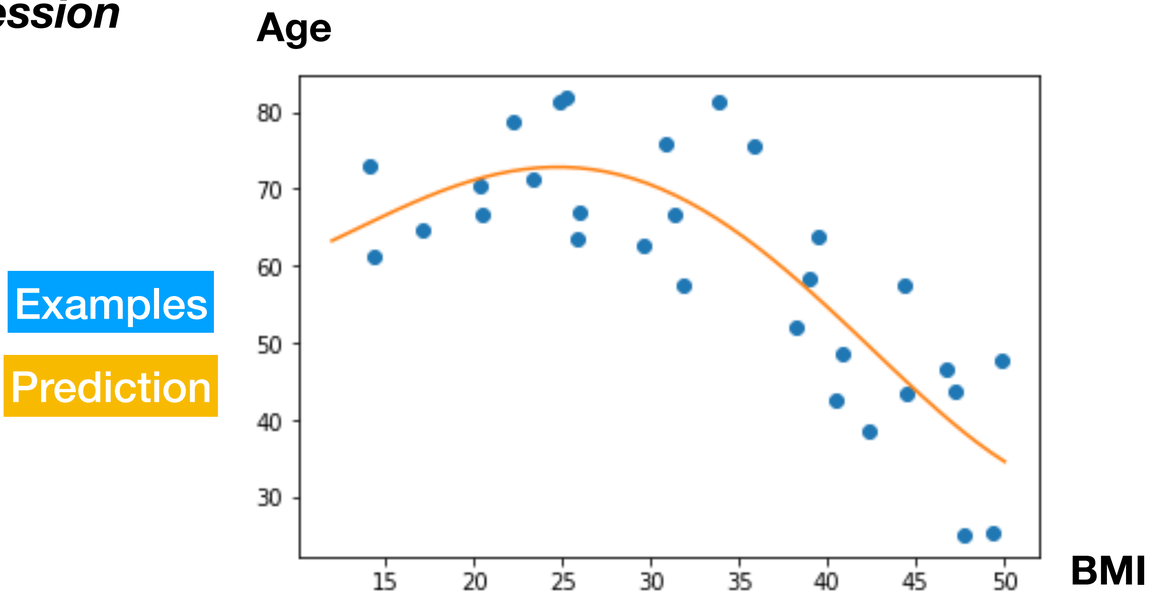
Classification

- In the last two sessions, we considered the task of ***regression***
- Given examples, predict a number
  - Age of Death
  - Price of a stock
  - Temperature of an object
  - ….



Classification

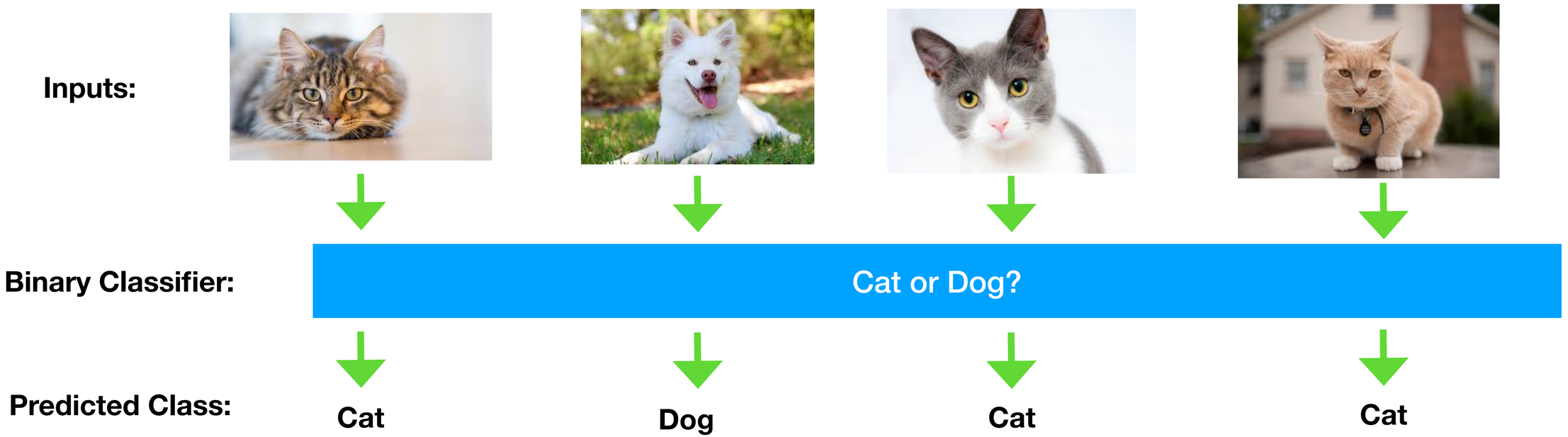
- In the last two sessions, we considered the task of ***regression***
- Given examples, predict a number
  - Age of Death
  - Price of a stock
  - Temperature of an object
  - ….
- In contrast, today, we are going to consider the task of ***classification***
  - Given examples, predict a **class**



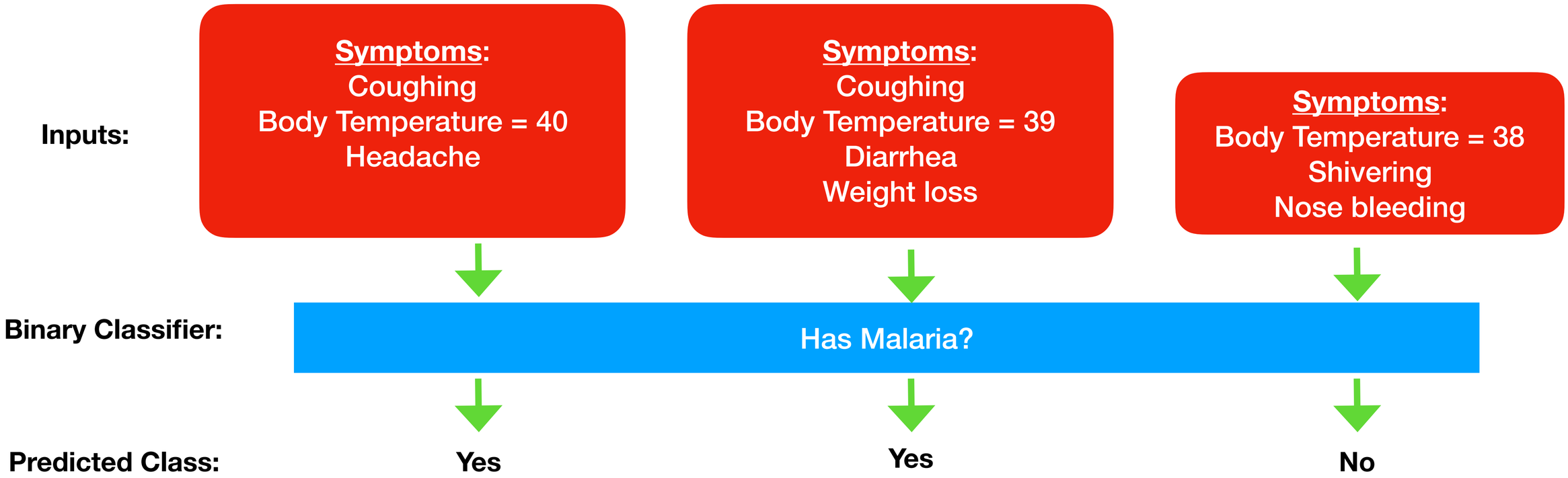
Classification

- The simple type of classification is called ***binary classification***
- **binary** means that we have 2 classes
- A binary classifier can answer questions of the type:
  - Yes or no?
  - Cat or Dog?
  - Red or Blue?
  - Healthy or Unhealthy?

Binary Classification



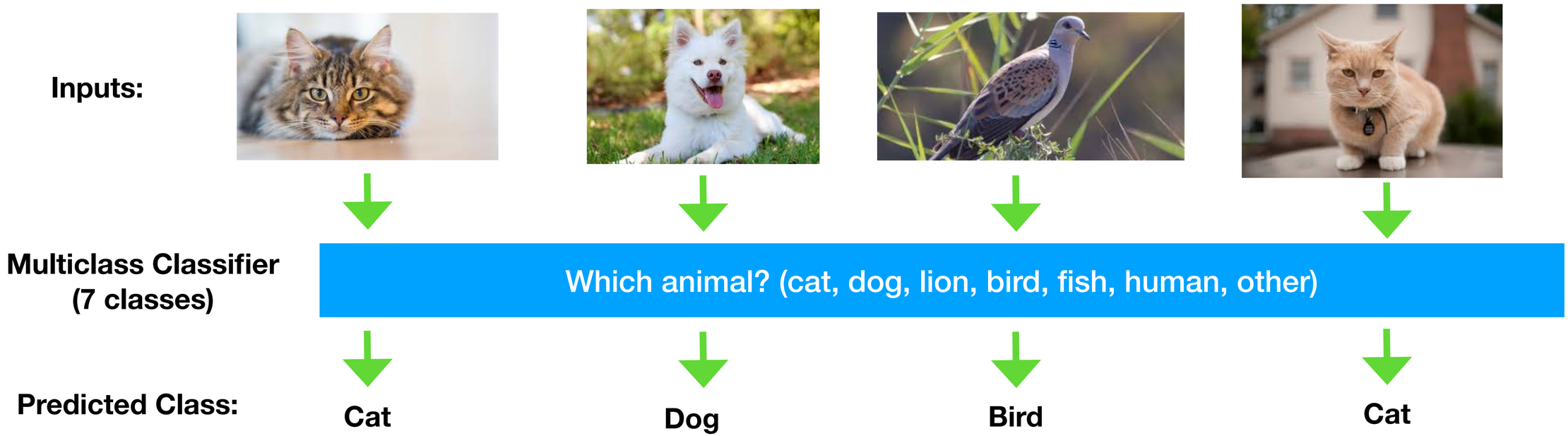
Binary Classification



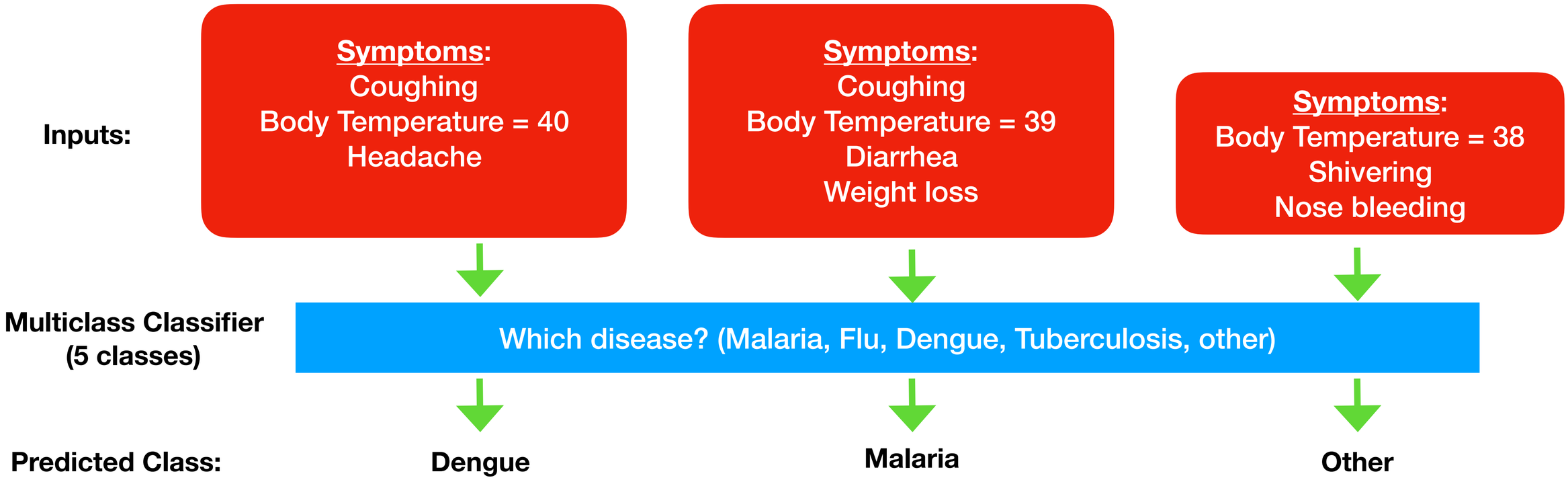
Multiclass Classification

- ***Multiclass classification*** is when we have more than 2 possible answers:
  - What animal is that? (cat, dog, lion, bird, fish, human, other) [7 classes]
  - Which disease is that? (Malaria, Flu, Dengue, Tuberculosis, other) [5 classes]
  - Which color is that? (red, blue, white, yellow, green, black) [6 classes]

Multiclass Classification

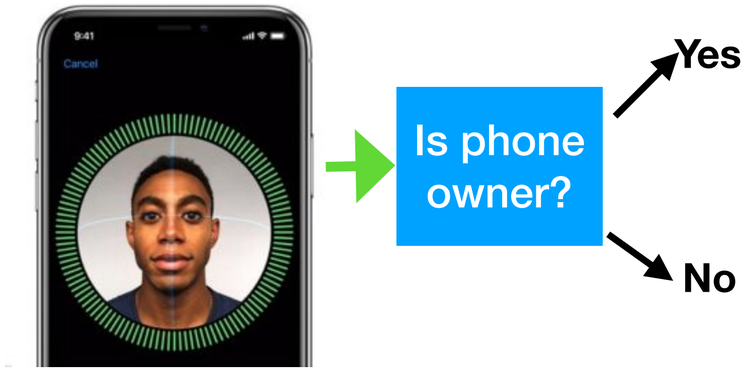


Binary Classification

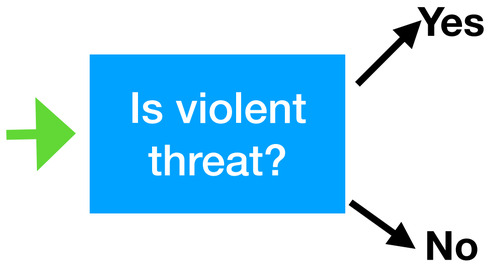


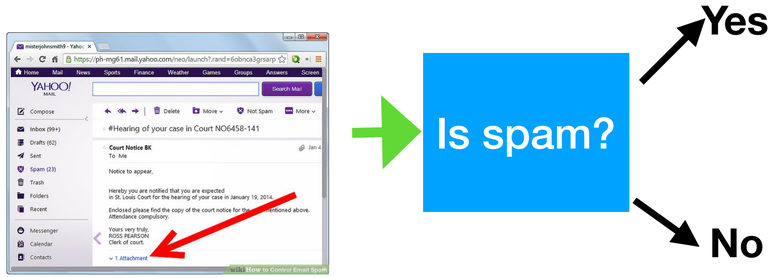
Classification

- Many interesting problems in AI can be seen as Classification problems
- Binary Classification:
  - Biometry-based identification (such as Face-id):
  - Spam detection
  - Automatic detection of use of bullying in social media
  - …



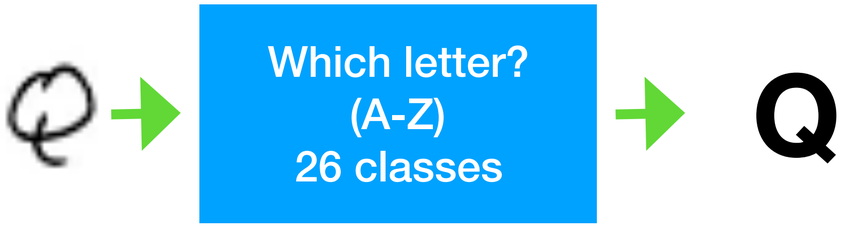
“I am going to punch you till your head explodes”

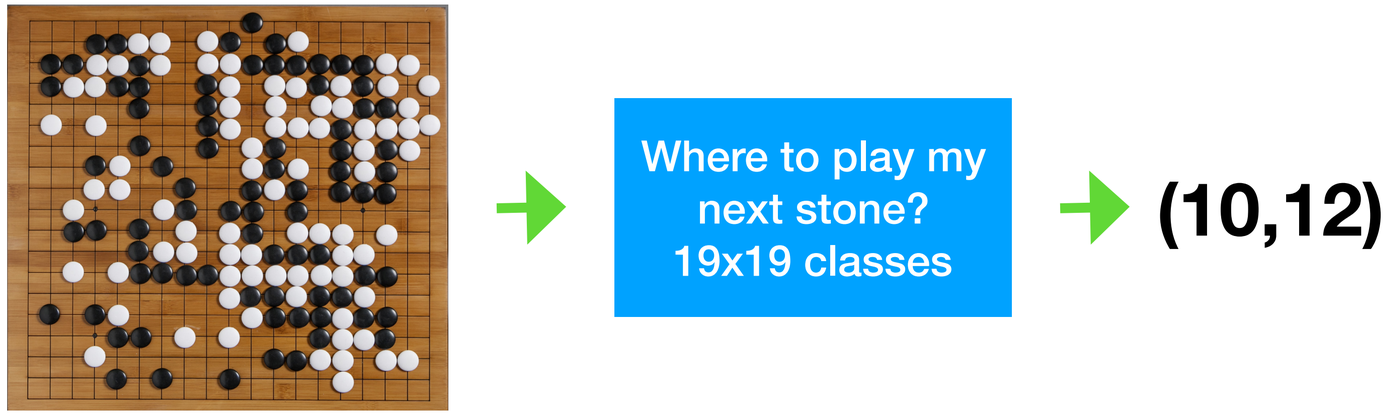




Classification

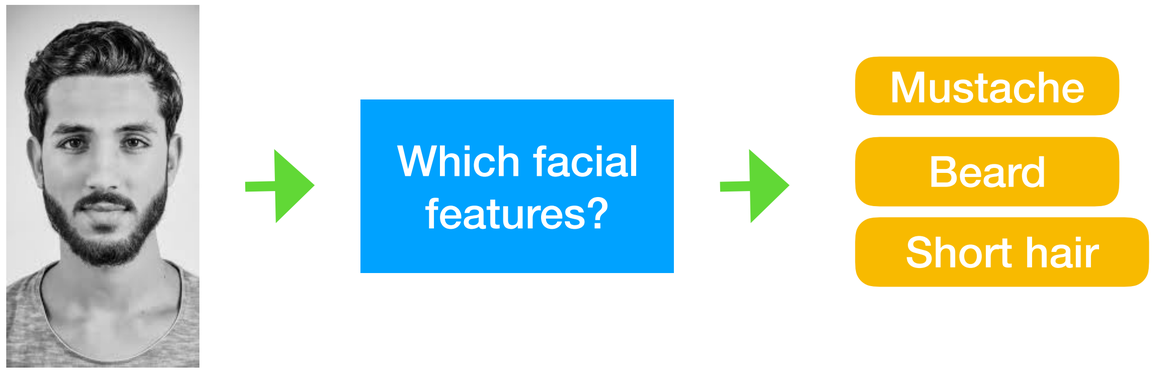
- Some interesting problems that can be seen as **multiclass** classification:
  - Optical Character Recognition (OCR)
  - AI for games
  - Tagging photos with people name
  - …..





Multiclass vs Multilabel

- In multiclass classification, classes are **exclusives**
  - For each input there is one and only one correct class
- If we want that an input may have more than one class, this is called ***multilabel classification***
  - But we will **not** consider this case for now



Binary Classification: Predicting vote

- As a practical example of **binary** classification, let us try to predict the **vote** of somebody given some features
  - For now, we only consider income as a feature
    - (consider it expressed in 1000$/year or 10万円/年 to get a concrete idea)
  - We only consider 2 parties for now: Left-wing party and Right-wing party
- We did a survey of 30 persons that accepted to answer us

	income	vote
0	39.0	L
1	30.0	L
2	47.0	L
3	69.0	R
4	52.0	R
5	110.0	R
6	…	….

- Now, given the income of somebody else, we want to **predict** for which party he is going to vote
- Useful task to:
  - Predict the result of an election (if we know the income of each person in the country, we can predict their vote efficiently)
  - Do efficient marketing on an individual

Binary Classification: Predicting vote

- Numbers are easier to use than letters in Machine Learning
- As a first step, we assign number 0 to class "*Left-Wing Party*" and number 1 to class "*Right-Wing Party*"
  - Which number you assign to which category will have no influence on prediction

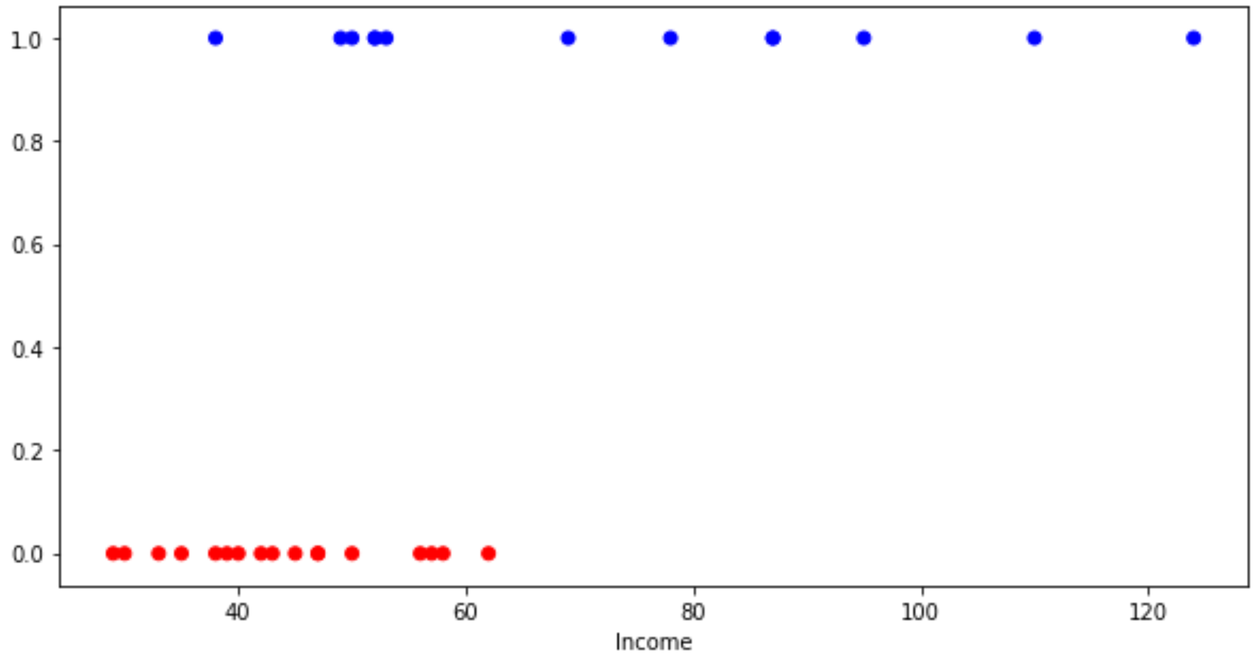
	income	vote
0	39.0	L
1	30.0	L
2	47.0	L
3	69.0	R
4	52.0	R
5	110.0	R
6	…	….

	income	vote
0	39.0	0
1	30.0	0
2	47.0	0
3	69.0	1
4	52.0	1
5	110.0	1
6	…	….

Binary Classification: Predicting vote

- Now, our 30 persons can be visualized like this:
  - Red: Left-wing party
  - Blue: Right-wing party

	income	vote
0	39.0	0
1	30.0	0
2	47.0	0
3	69.0	1
4	52.0	1
5	110.0	1
6	…	….



Binary Classification: Predicting vote

- How to predict a class?
- We are going to give a **score** to the possibility that a given voter is of **class 1** (ie. vote for the *right-wing party*)
- Because it is a **binary classification**, we do not need to compute a score for the other class

	income	vote
0	39.0	0
1	30.0	0
2	47.0	0
3	69.0	1
4	52.0	1
5	110.0	1
6	…	….

- The score could be any function of the features
- But this time, we will use a linear function again:

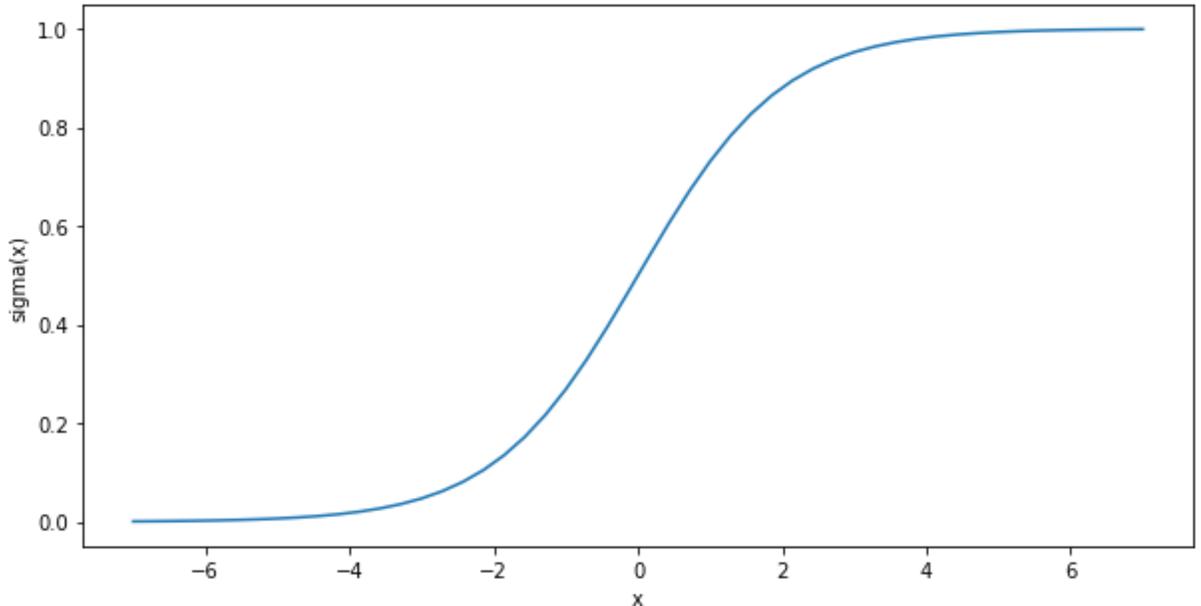
$$score(income) = \theta_0 + \theta_1 \times income$$

The Logistic function

- We will then apply the ***logistic function*** to the score

- The logistic function (also called ***sigma function***) is defined by:

$$\sigma(x) = \frac{1}{1 + exp(-x)}$$



- If *score* is very large, σ(*score*) will be close to 1
- If *score = 0 then* σ(*score*) = 0.5
- If *score* is very negative, σ(*score*) will be close to 1

Binary Classification: Predicting vote

	income	vote
0	39.0	0
1	30.0	0
2	47.0	0
3	69.0	1
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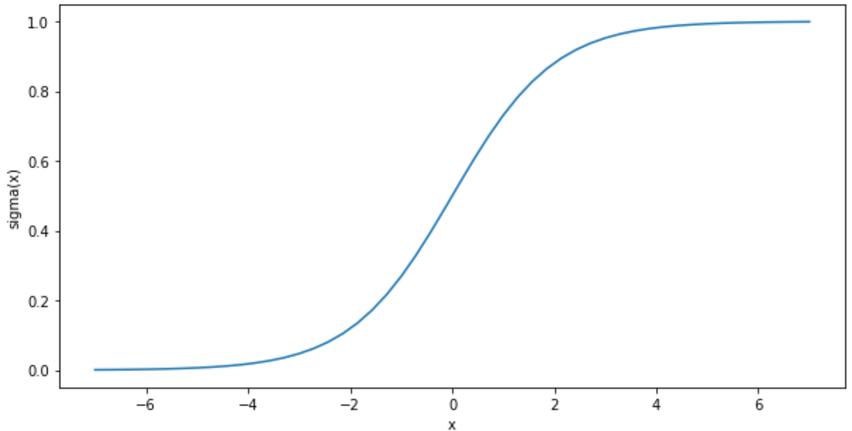


- Therefore, our classification model is like this:

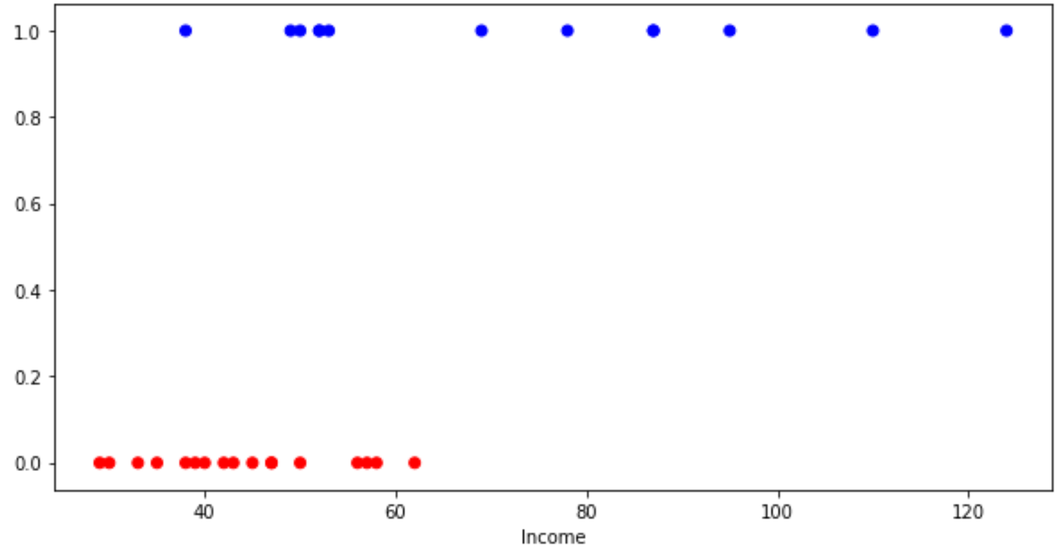
$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

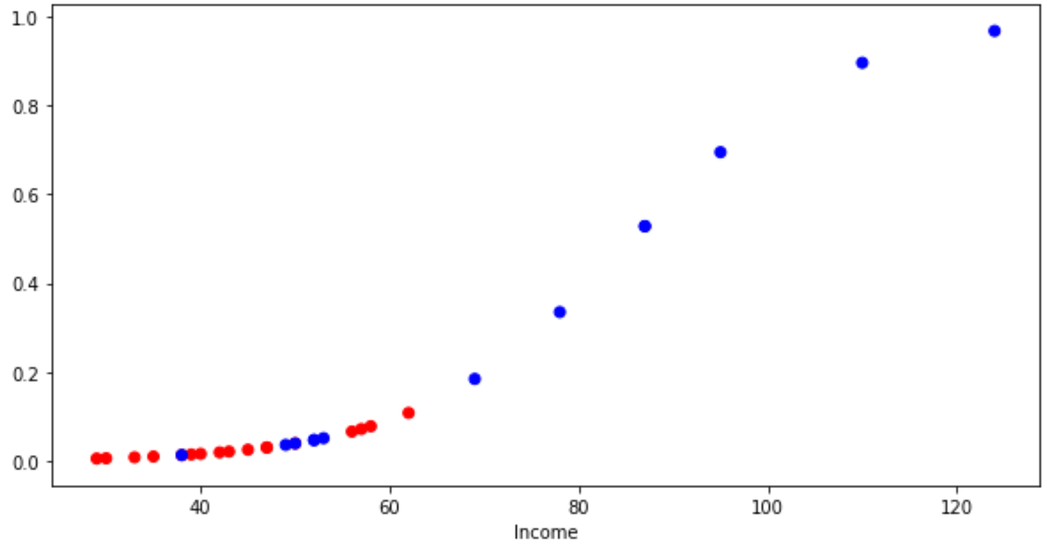
- In practice, the predicted ***V*** will not be equal to **0** or **1**
- The predicted ***V*** is a value between **0** and **1**
- We interpret ***V*** as the probability that a given voter will vote for the right wing party
- We can also interpret ***V*** as the confidence of the model that a given voter will vote for the right wing party

Difference between prediction and data

- Our examples represent binary values
- Either **0** or **1**

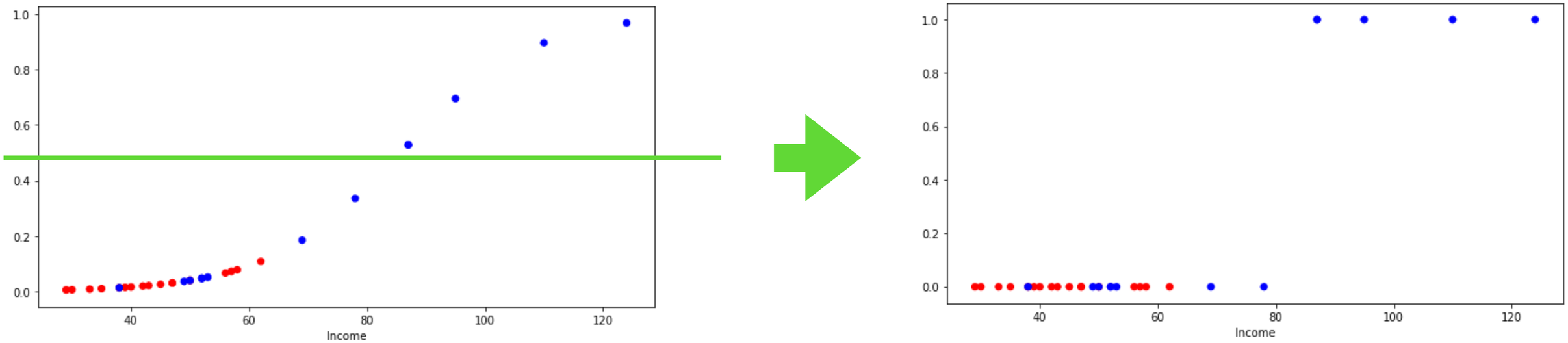


- Our predictions are probabilities <u>between</u> **0** or **1**
- We <u>want</u> to predict the data: either **0** or **1**



Difference between prediction and data

- Our predictions are probabilities between **0** or **1**
- We want to predict the data: either **0** or **1**
- -> We choose a decision boundary
  - (usually we check if predicted value is higher or lower than 0.5)



Our model predictions

- Finally, we now have predictions from our model

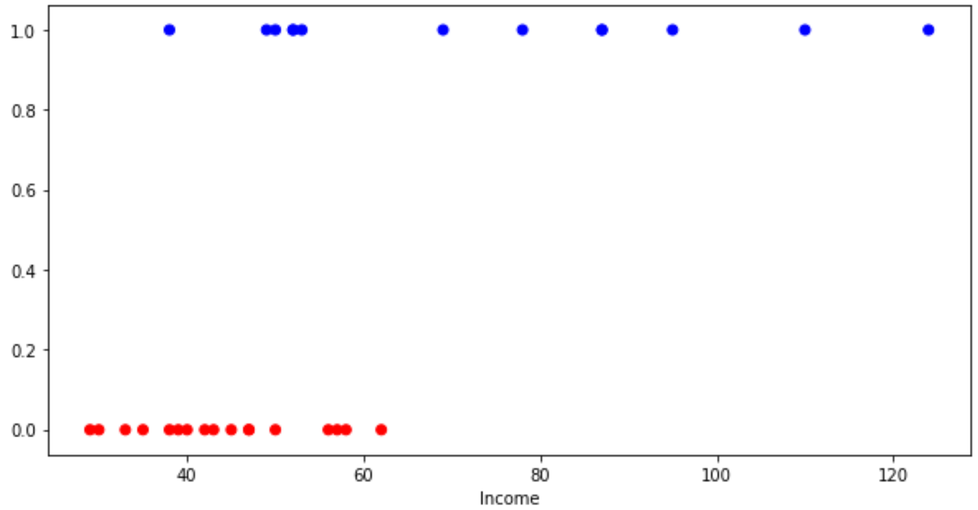
$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

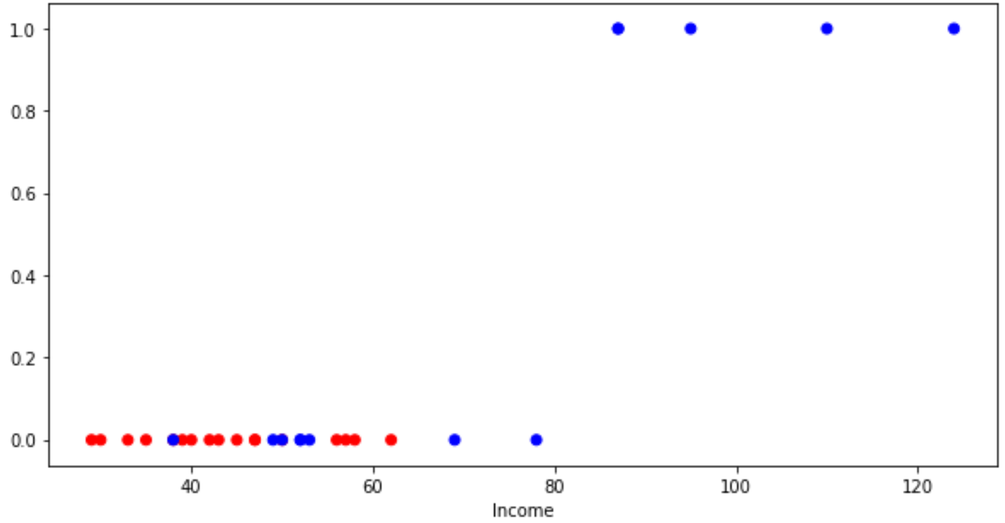
Class = 1 if V >0.5 else 0

- Remaining question: **how to find the parameters θ** ?

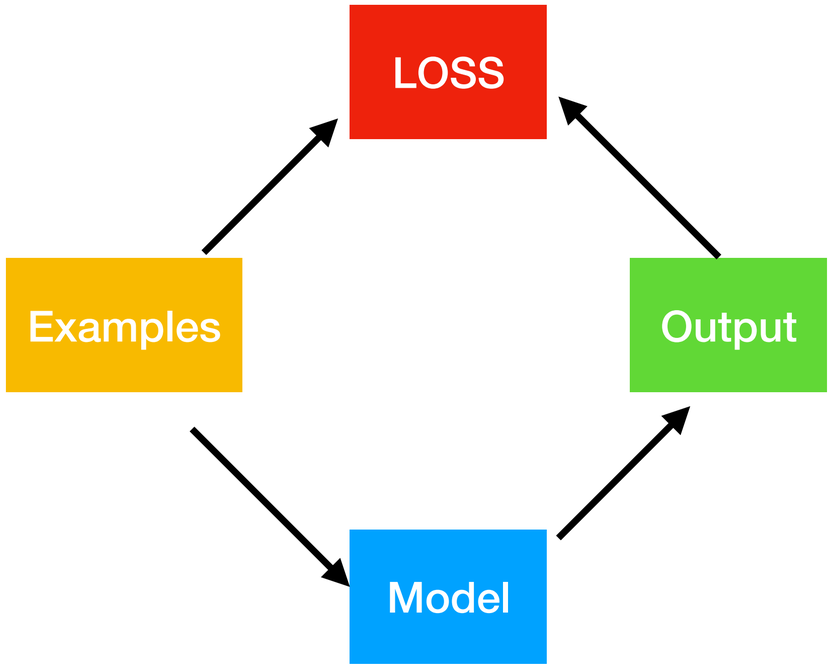
Example data

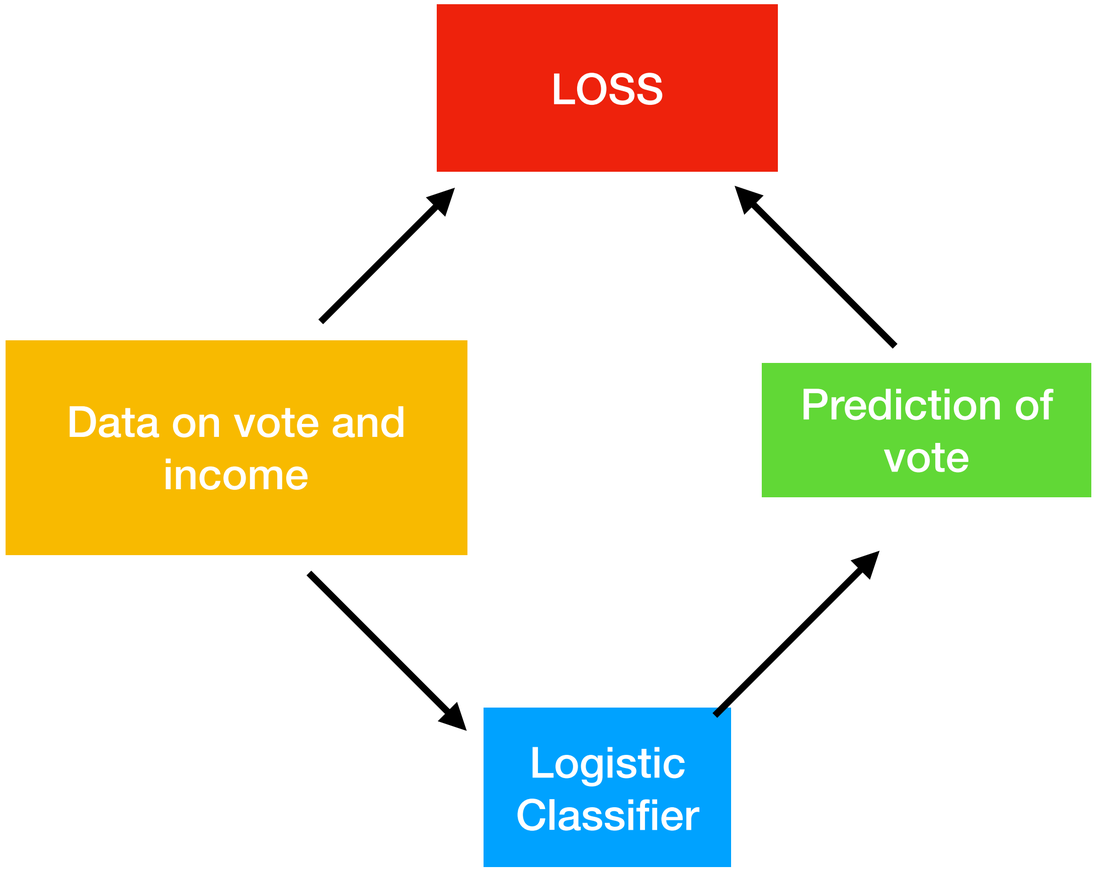


Our model predictions



How to find the parameters θ?





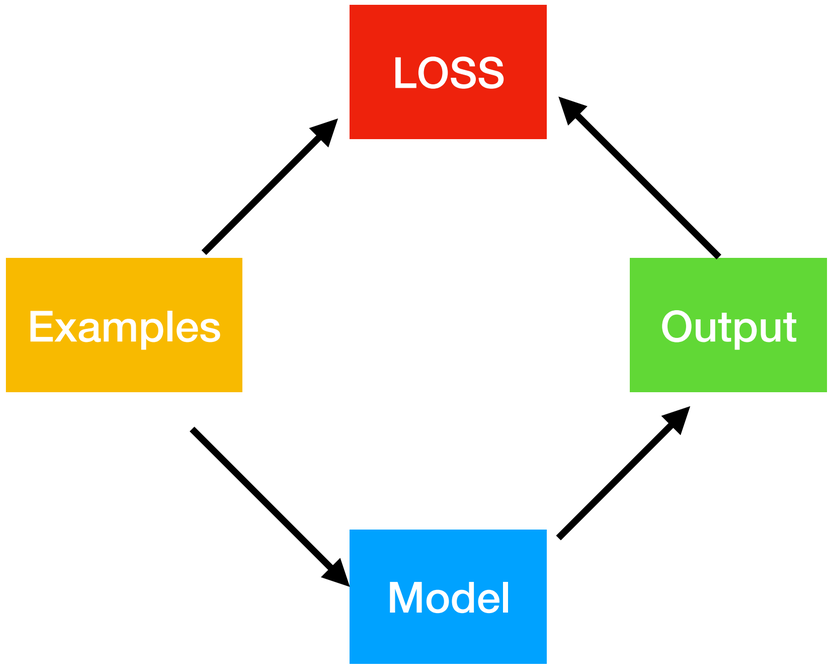
We have some examples

We have a model with some parameters

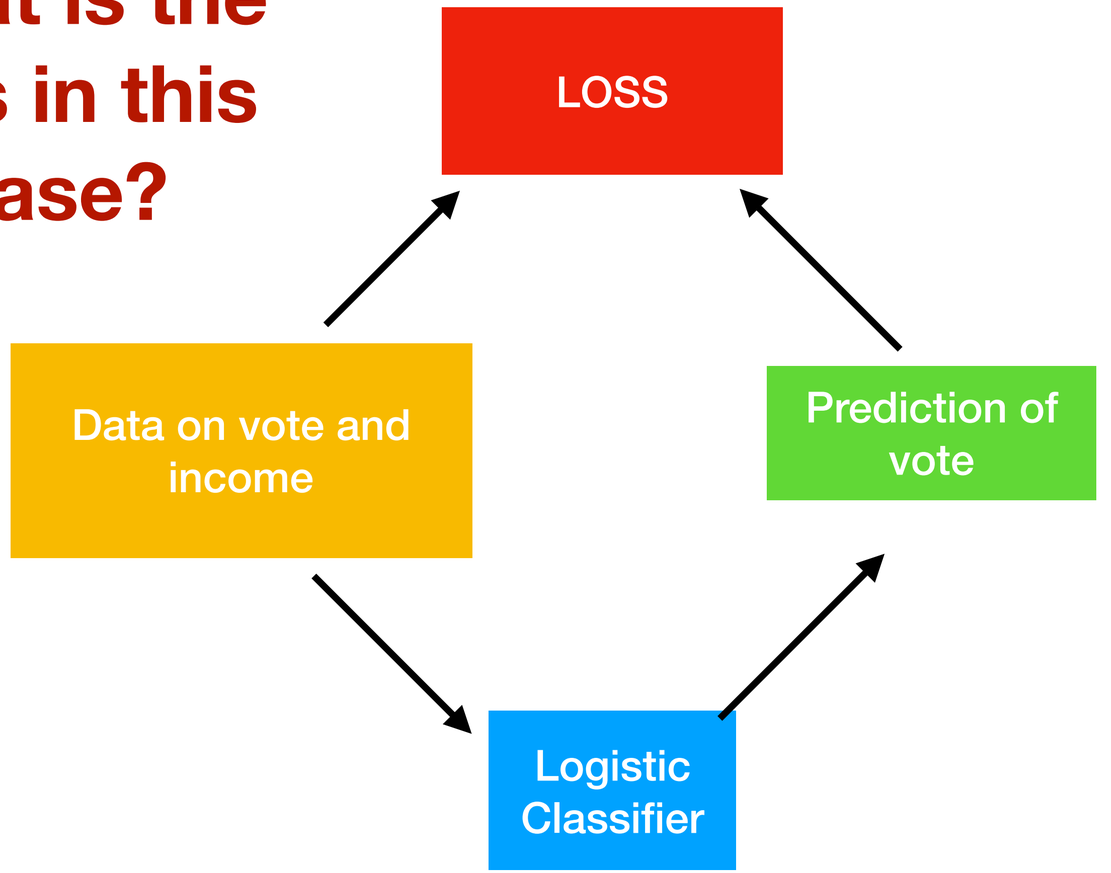
We have a loss function that compute the difference between the example and the prediction of our model

We minimize the loss to obtain the best parameters for our model

How to find the parameters θ?



What is the loss in this case?



We have some examples

We have a model with some parameters

We have a loss function that compute the difference between the example and the prediction of our model

We minimize the loss to obtain the best parameters for our model

What is the loss we should use?

- We need a loss that tells us how bad are the predictions given the examples

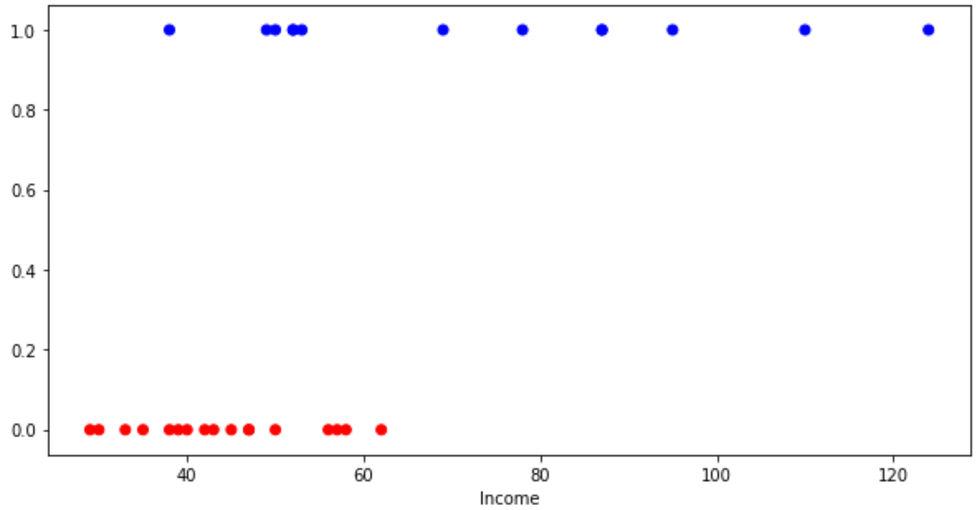
- One possibility: we count how many predictions were wrong

7 right-wing voters were predicted to vote left
0 left-wing voters were predicted to vote right
23 out of 30 predictions are correct
<u>Accuracy</u>: 23/30
<u>Loss</u>: 7/30

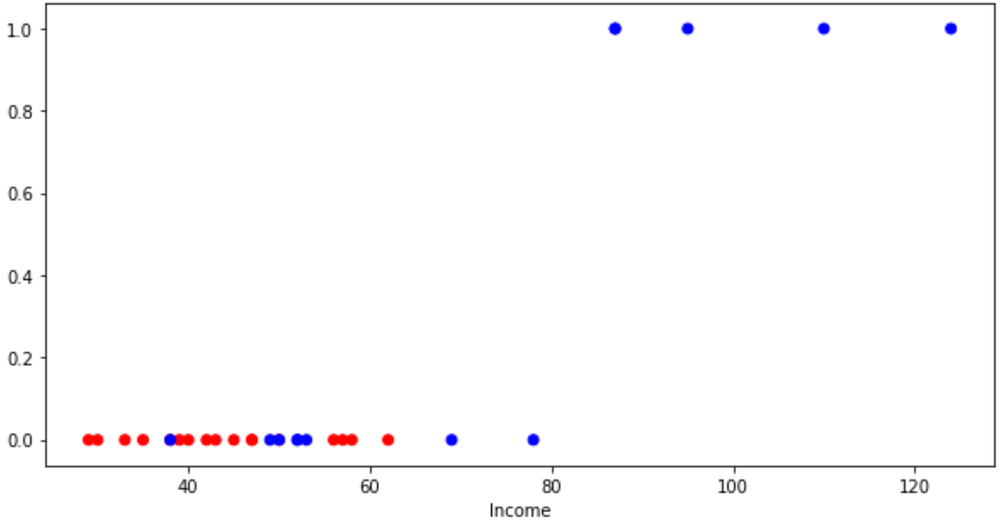
- Intuitively, it seems a good way to evaluate the model

- Unfortunately, it is **<u>not a good loss</u>** for learning parameters

Example data



Our model predictions



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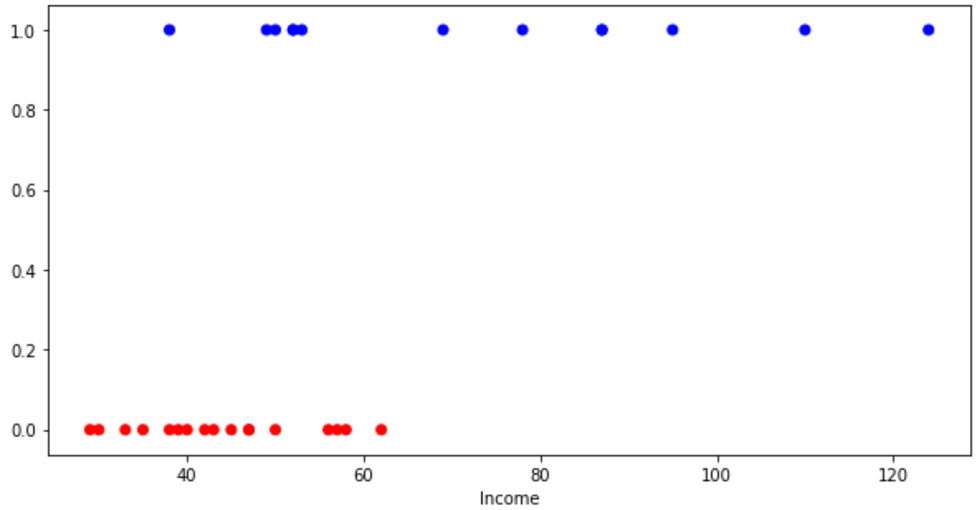

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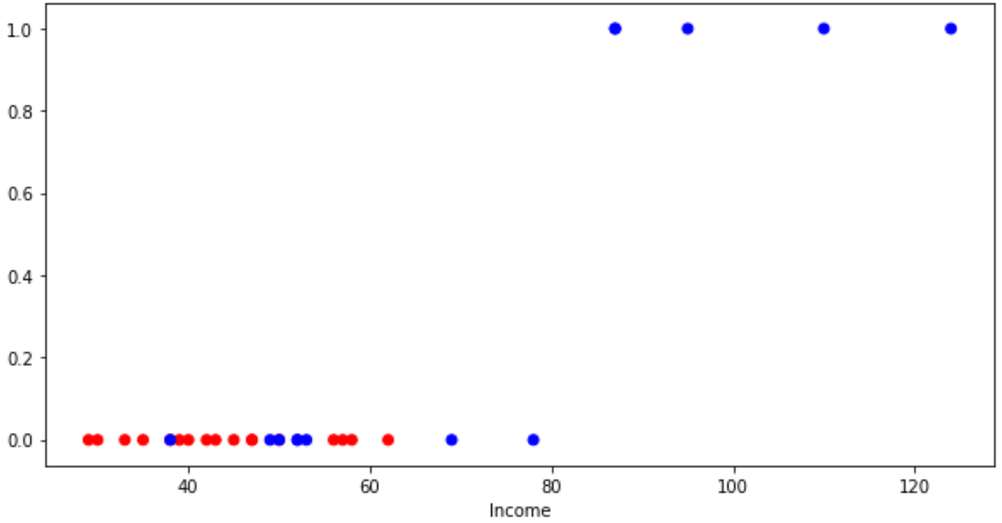
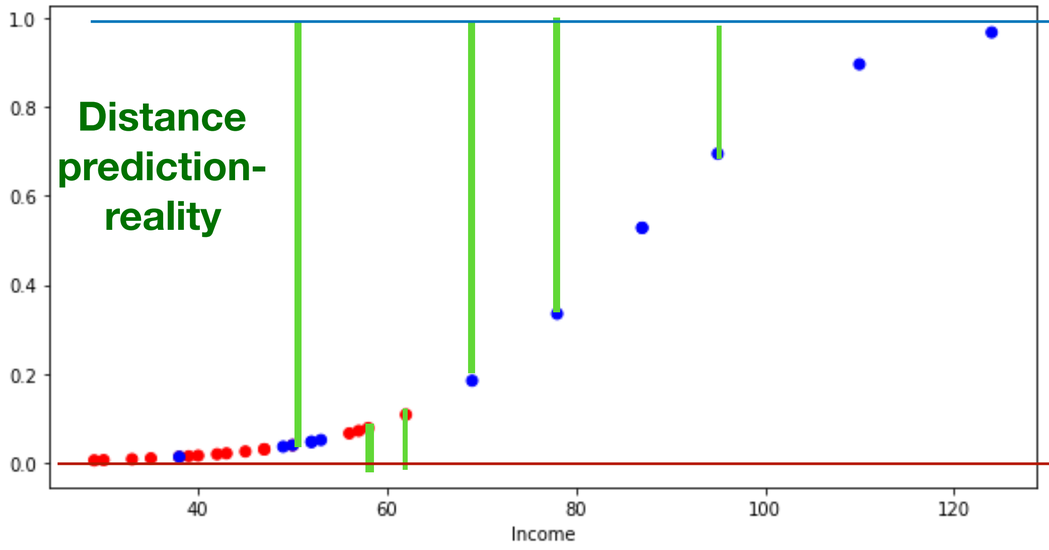


- Intuitively, it seems a good way to evaluate the model
- Unfortunately, it is **not a good loss** for learning parameters
- Because this loss is not a continuous function of the parameters θ
- We cannot compute a gradient -> we cannot use gradient descent

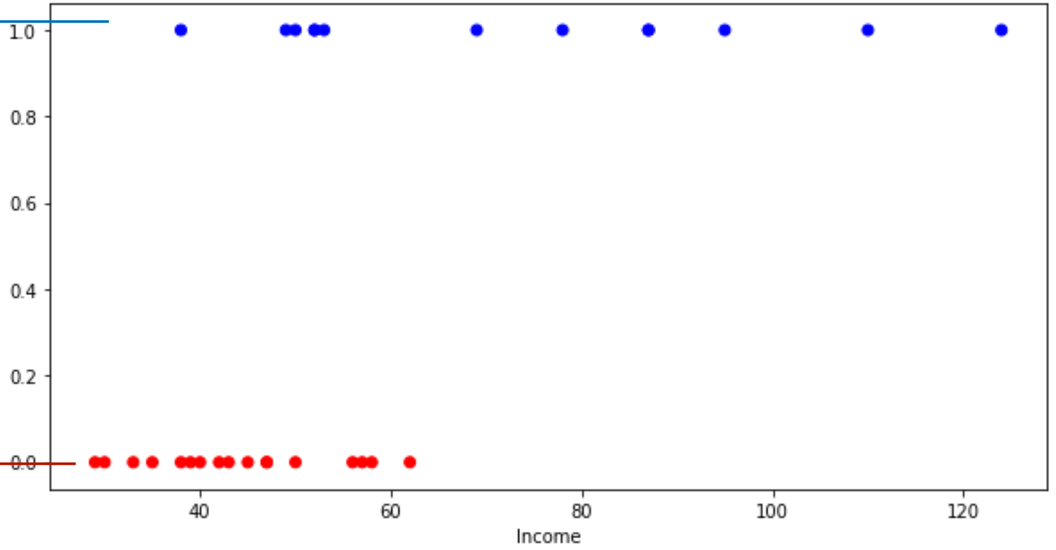
What is the loss we should use?

- How about the ***Mean Squared Distance***?
- If we use the predicted probability instead of the predicted class $V_{model} = \sigma(score)$
- Then the same situation as with regression: we compute the distance between the prediction and the real value
- In theory it works; **in practice, it does not work**
- The Mean Squared Distance loss of a logistic classifier is very difficult to minimize in practice (for technical reasons we will not discuss here)

Model's Prediction

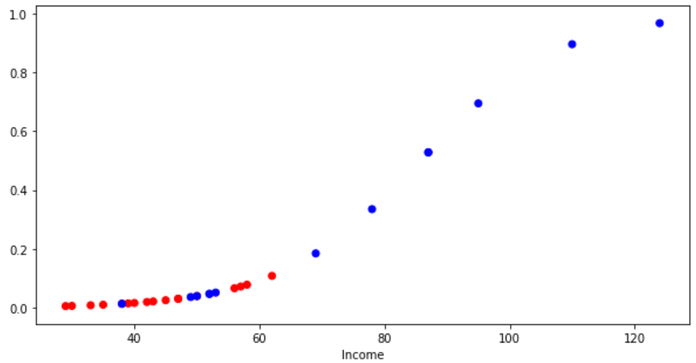


Example data



What is the loss we should use?

$$V_{model} = \sigma(score)$$

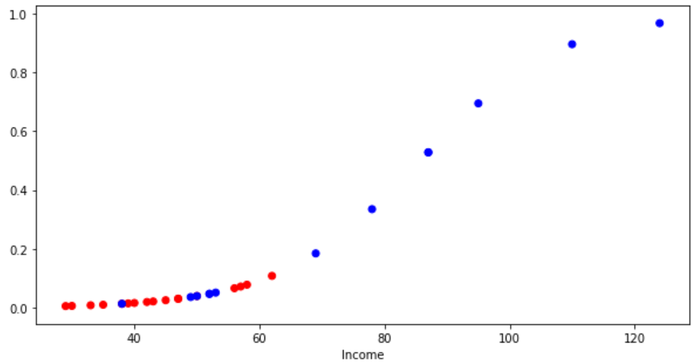


- We will use something called the ***cross-entropy loss***
- It is defined as follow
- For each example:
  - If the correct class is **1**, then the ***cost*** of the prediction **V** is -log(**V**)
  - If the correct class is **0**, then the ***cost*** of the prediction **V** is -log(1-**V**)
- Then we average the cost over each example

$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

The cross entropy loss

$$V_{model} = \sigma(score)$$



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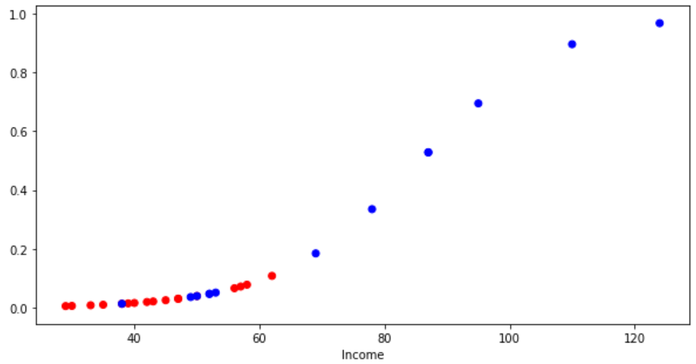
- Why it might work?

The cross entropy loss

$$V_{model} = \sigma(score)$$



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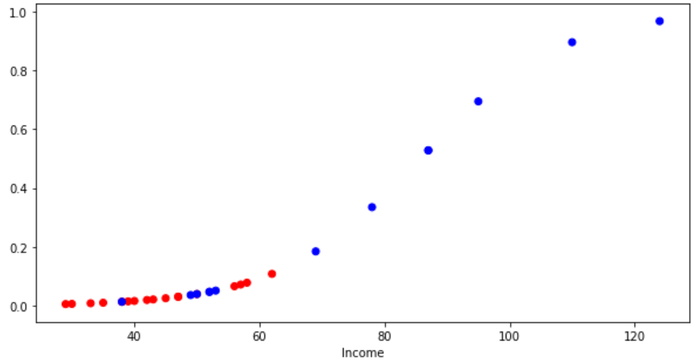
- Then we average the cost over each example

$$V_{model} = \sigma(score)$$

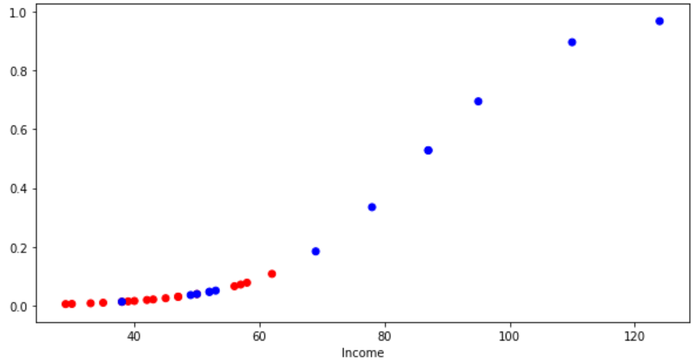


- If, for a given person, **V**=**1**: the model say that this person will **certainly** vote for the right wing party
  - If the given person actually voted for the right-wing party (correct class is **1**), then the cost is -log(**1**) = 0
  - If the given person actually voted for the left-wing party (correct class is **0**), then the cost is -log(**1-1**) = ∞

The cross entropy loss

- For each example:
  - If the <u>correct class</u> is **1**, then the ***cost*** of the prediction **V** is -log(**V**)
  - If the <u>correct class</u> is **0**, then the ***cost*** of the prediction **V** is -log(1-**V**)
- Then we average the cost over each example

$$V_{model} = \sigma(score)$$

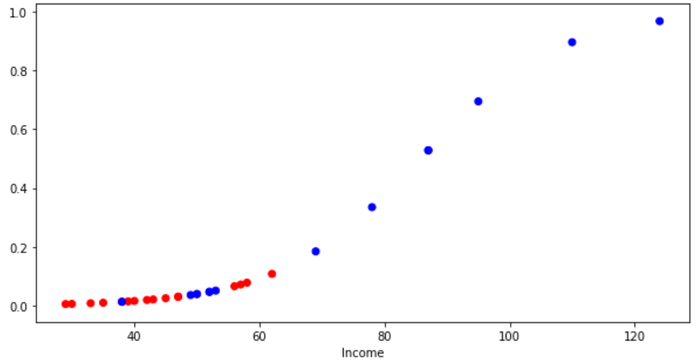


- If, for a given person, **V**=**0**: the model say that this person will **<u>certainly</u>** vote for the left wing party
  - If the given person <u>actually</u> voted for the right-wing party (correct class is **1**), then the cost is -log(**0**) = ∞
  - If the given person <u>actually</u> voted for the left-wing party (correct class is **0**), then the cost is -log(**1-0**) = 0

The cross entropy loss

- For each example:
  - If the correct class is **1**, then the ***cost*** of the prediction **V** is -log(**V**)
  - If the correct class is **0**, then the ***cost*** of the prediction **V** is -log(1-**V**)
- Then we average the cost over each example

$$V_{model} = \sigma(score)$$



- If, for a given person, **V**=**0.5**: the model say that there is an equal probability that the voter will vote for any party
  - If the given person actually voted for the right-wing party (correct class is **1**), then the cost is -log(**0.5**) = 0.69
  - If the given person actually voted for the left-wing party (correct class is **0**), then the cost is -log(**1-0.5**) = 0.69

The cross entropy loss

- In short, the cost will be zero only if the model made a **very confident** correct prediction
- The cost will be very large if the model made a **very confident** incorrect prediction
- If the model made prediction with little confidence (V is close to 0.5), the cost will be moderate whatever the correct answer was
- If the model always make correct predictions with high confidence, the average of the cost will be close to zero

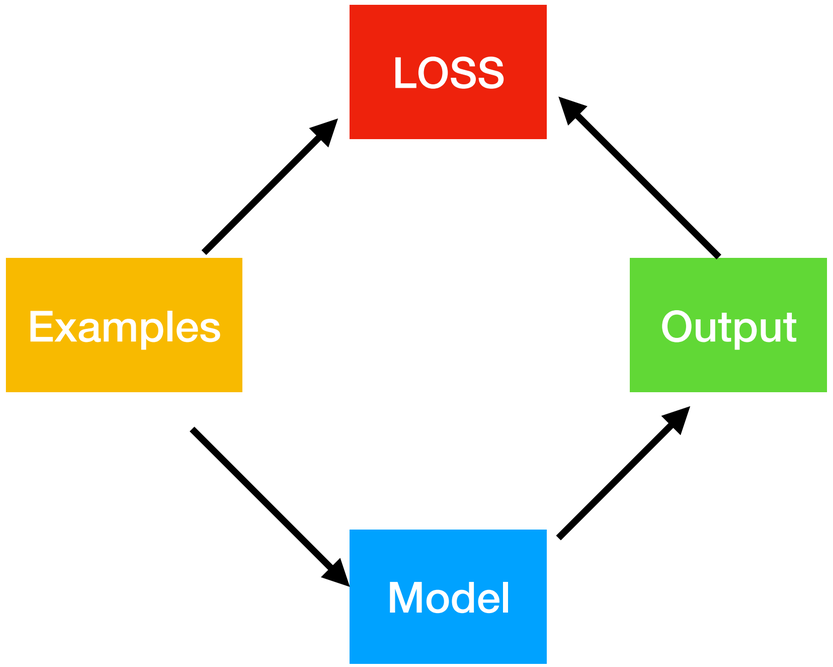
$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

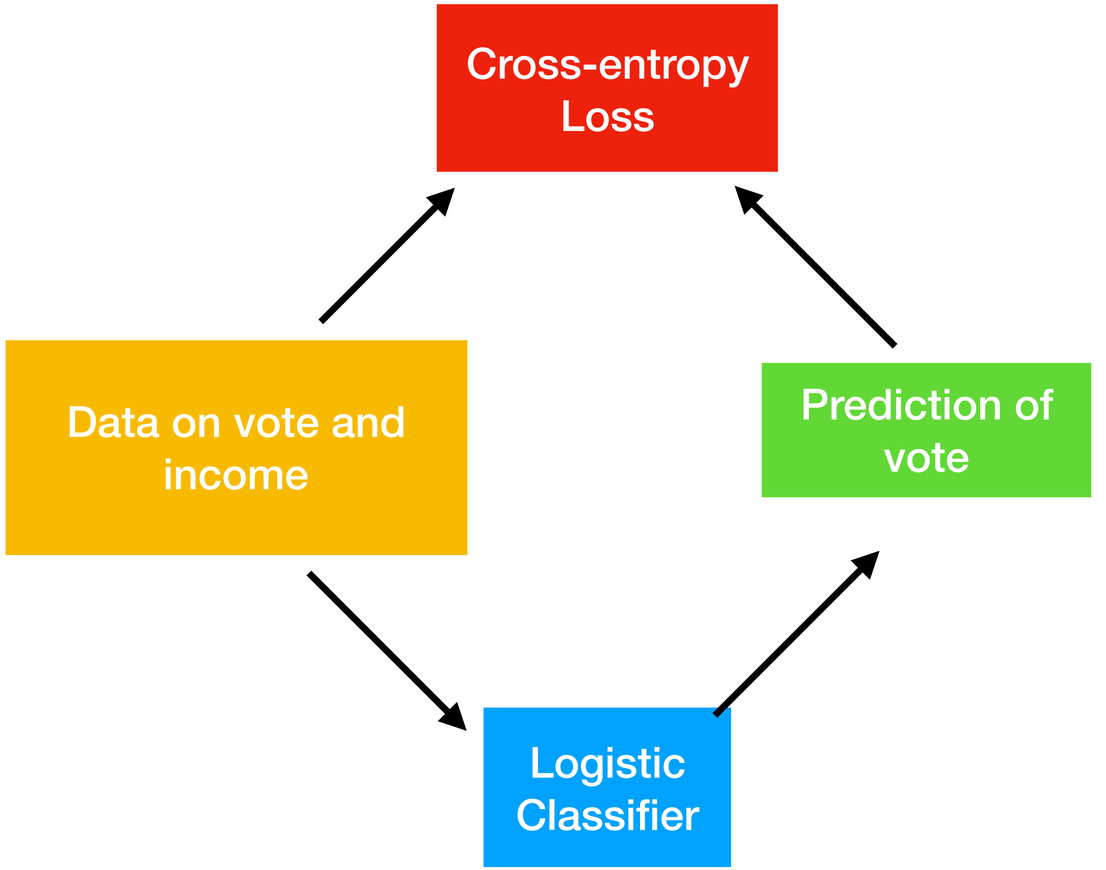
The cross entropy loss

- From this it follows that the CrossEntropyLoss is a good loss for our setting
- If we minimize it, we will find a good model
- It is a continuous (and smooth) function of the parameters
- It is easy to minimize in practice

$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

How to find the parameters θ?





We have some examples

We have a model with some parameters

We have a loss function that compute the difference between the example and the prediction of our model

We minimize the loss to obtain the best parameters for our model

Minimizing the cross-entropy loss

- As before, to minimize the loss, we need to compute its gradient with respect to the parameters

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

We can find that:

$$\frac{\partial}{\partial \theta_0} CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} (V_{model}(income_i) - V_i)$$

$$\frac{\partial}{\partial \theta_1} CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} (V_{model}(income_i) - V_i) \times income_i$$

Minimizing the cross-entropy loss

- As before, to minimize the loss, we need to compute its gradient with respect to the parameters

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

We can find that:

$$\frac{\partial}{\partial \theta_0} CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} error_i$$

$$\frac{\partial}{\partial \theta_1} CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} error_i \times income_i$$

With:

$$error_i = V_{model}(income_i) - V_i$$

$$grad \cdot CEL = [\frac{\partial}{\partial \theta_0} CEL, \frac{\partial}{\partial \theta_1} CEL] = [\frac{1}{N}\sum_{i=1}^{N} error_i, \frac{1}{N}\sum_{i=1}^{N} error_i \times income_i]$$

Our data

	income	vote
0	39.0	0
1	30.0	0
2	47.0	0
3	69.0	1
4	52.0	1
5	110.0	1
6	…	….

Minimizing the cross-entropy loss

- Now that we know the gradient of the loss, we can minimize the loss to find the best parameters θ
- To make things a bit less abstract, let’s try to do it in practice on a small example

Example

Example Data

	income	vote
1	40	0
2	70	1
3	20	0

- Suppose these parameters: $\theta_0 = -8$ $\theta_1 = 0.1$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$\sigma(x) = \frac{1}{1 + exp(-x)}$$

	income	vote	Score	Prediction V	Predicted Class	Cost
1	40	0				
2	70	1				
3	20	0				

Example

Example Data

	income	vote
1	40	0
2	70	1
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- Suppose these parameters: $\theta_0 = -8$ $\theta_1 = 0.1$

$$score(income) = \theta_0 + \theta_1 \times income$$

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$$\sigma(x) = \frac{1}{1 + exp(-x)}$$

	income	vote	Score	Prediction V	Predicted Class	Cost
1	40	0	-4	0.018	0	
2	70	1	-1	0.269	0	
3	20	0	-6	0.002	0	

Example

Example Data

	income	vote
1	40	0
2	70	1
3	20	0

- Suppose these parameters: $\theta_0 = -8$ $\theta_1 = 0.1$

$$score(income) = \theta_0 + \theta_1 \times income$$

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$$\sigma(x) = \frac{1}{1 + exp(-x)}$$

	income	vote	Score	Prediction V	Predicted Class	Cost
1	40	0	-4	0.018	0	
2	70	1	-1	0.269	0	
3	20	0	-6	0.002	0	

- if the correct class is **1**, then the ***cost*** of the prediction **V** is -log(**V**)
- If the correct class is **0**, then the ***cost*** of the prediction **V** is -log(1-**V**)

$$CrossEntropyLoss = \frac{1}{N} \sum_{i=1}^{N} cost_i$$

Example

Example Data

	income	vote
1	40	0
2	70	1
3	20	0

- Suppose these parameters: $\theta_0 = -8$ $\theta_1 = 0.1$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$\sigma(x) = \frac{1}{1 + exp(-x)}$$

	income	vote	Score	Prediction V	Predicted Class	Cost
1	40	0	-4	0.018	0	0.018
2	70	1	-1	0.269	0	1.31
3	20	0	-6	0.002	0	0.002

- if the correct class is **1**, then the ***cost*** of the prediction **V** is -log(**V**)
- If the correct class is **0**, then the ***cost*** of the prediction **V** is -log(1-**V**)

$$CrossEntropy = \frac{1}{3}(0.018 + 1.31 + 0.002) = 0.44$$

Example

Example Data

	income	vote
1	40	0
2	70	1
3	20	0

$$\theta_0 = -8 \qquad \theta_1 = 0.1$$

- Now suppose we want to improve our parameters
- Let us do one iteration of gradient descent

	income	vote	Score	Prediction V	Predicted Class	Cost
1	40	0	-4	0.018	0	0.018
2	70	1	-1	0.269	0	1.31
3	20	0	-6	0.002	0	0.002

$$CrossEntropy = \frac{1}{3}(0.018 + 1.31 + 0.002) = 0.44$$

Computing the gradient

$$\theta_0 = -8 \qquad \theta_1 = 0.1$$

	income	vote	Score	Prediction V	Predicted Class	Cost	Error
1	40	0	-4	0.018	0	0.018	
2	70	1	-1	0.269	0	1.31	
3	20	0	-6	0.002	0	0.002	

$$error_i = V_{model}(income_i) - V_i$$

$$grad \cdot CEL = [\frac{\partial}{\partial \theta_0} CEL, \frac{\partial}{\partial \theta_1} CEL] = [\frac{1}{N} \sum_{i=1}^{N} error_i, \frac{1}{N} \sum_{i=1}^{N} error_i \times income_i]$$

Computing the gradient

$$\theta_0 = -8 \qquad \theta_1 = 0.1$$

	income	vote	Score	Prediction V	Predicted Class	Cost	Error
1	40	0	-4	0.018	0	0.018	0.018
2	70	1	-1	0.269	0	1.31	-0.731
3	20	0	-6	0.002	0	0.002	0.002

$$error_i = V_{model}(income_i) - V_i$$

$$grad \cdot CEL = [\frac{1}{N}\sum_{i=1}^{N} error_i, \frac{1}{N}\sum_{i=1}^{N} error_i \times income_i]$$

$$= \frac{1}{3}[0.018 - 0.731 + 0.002, 0.018 \times 40 - 0.731 \times 70 + 0.002 \times 20] = [-0.23, -16.8]$$

Doing one iteration of gradient descent

	income	vote	Score	Prediction V	Predicted Class	Cost	Error
1	40	0	-4	0.018	0	0.018	0.018
2	70	1	-1	0.269	0	1.31	-0.731
3	20	0	-6	0.002	0	0.002	0.002

$\theta_0 = -8 \qquad \theta_1 = 0.1$

$grad \cdot CrossEntropy = [-0.23, -16.8]$

- Let us do one iteration of gradient descent with lr = 0.005
- What are new parameters θ?

Doing one iteration of gradient descent

	income	vote	Score	Prediction V	Predicted Class	Cost	Error
1	40	0	-4	0.018	0	0.018	0.018
2	70	1	-1	0.269	0	1.31	-0.731
3	20	0	-6	0.002	0	0.002	0.002

$\theta_0 = -8 \qquad \theta_1 = 0.1$

$grad \cdot CrossEntropy = [-0.23, -16.8]$

- Let us do one iteration of gradient descent with lr = 0.005
- What are new parameters θ?

$\theta_0 = -8 - 0.005 \times -0.23 = -7.999$ $\qquad$ $\theta_1 = 0.1 - 0.005 \times -16.8 = 0.184$

Trying the new parameters

Example Data

	income	vote
1	40	0
2	70	1
3	20	0

- Let us try these new parameters

$$\theta_0 = -7.999$$

$$\theta_1 = 0.184$$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$\sigma(x) = \frac{1}{1 + exp(-x)}$$

	income	vote	Score	Prediction V	Predicted Class	Cost
1	40	0				
2	70	1				
3	20	0				

Trying the new parameters

Example Data

	income	vote
1	40	0
2	70	1
3	20	0

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$$\theta_0 = -7.999$$

$$\theta_1 = 0.184$$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$\sigma(x) = \frac{1}{1 + exp(-x)}$$

	income	vote	Score	Prediction V	Predicted Class	Cost
1	40	0	-0.639	0.34	0	
2	70	1	4.88	0.99	1	
3	20	0	-4.319	0.01	0	

Trying the new parameters

Example Data

	income	vote
1	40	0
2	70	1
3	20	0

- Let us try these new parameters

$$\theta_0 = -7.999$$

$$\theta_1 = 0.184$$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$\sigma(x) = \frac{1}{1 + exp(-x)}$$

	income	vote	Score	Prediction V	Predicted Class	Cost
1	40	0	-0.639	0.34	0	0.4
2	70	1	4.88	0.99	1	0.01
3	20	0	-4.319	0.01	0	0.013

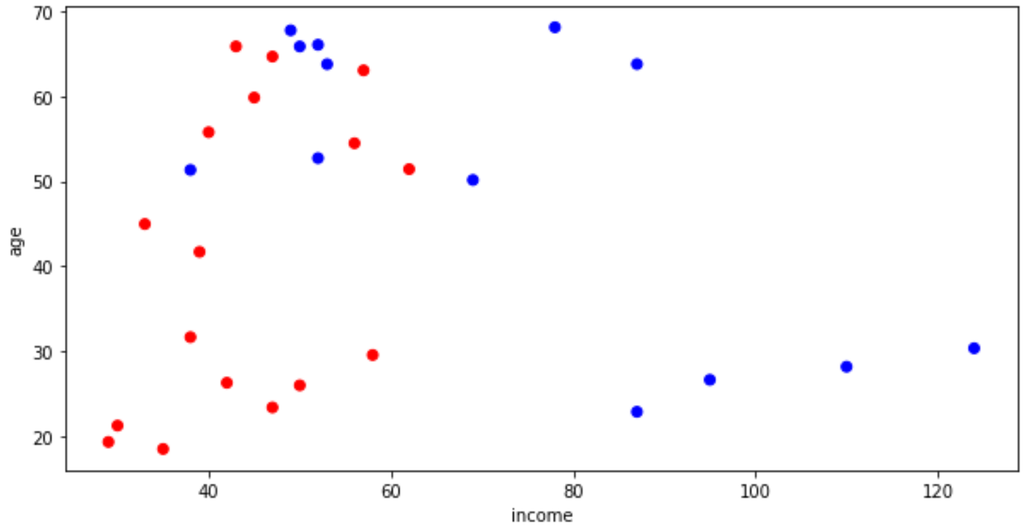
$$CrossEntropy = \frac{1}{3}(0.4 + 0.01 + 0.013) = 0.15$$

Cross entropy decreased from 0.43 to 0.15

Classification with more than one feature

- Let us now suppose that, on top of **income**, we have information about **age**
- How do we change our Logistic model to take into account the new information?

	income	age	vote
0	39.0	42.0	L
1	30.0	21.0	L
2	47.0	65.0	L
3	69.0	50.0	R
4	52.0	53.0	R
5	110.0	28.0	R
…	…	…	…



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2	47.0	65.0	L
3	69.0	50.0	R
4	52.0	53.0	R
5	110.0	28.0	R
...	...	...	...

EASY! We just add a new parameter to the score function:

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$score(income, age) = \theta_0 + \theta_1 \times income + \theta_2 \times age$$

$$V_{model} = \sigma(score)$$

Classification with more than one feature

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$$V_{model} = \sigma(score)$$

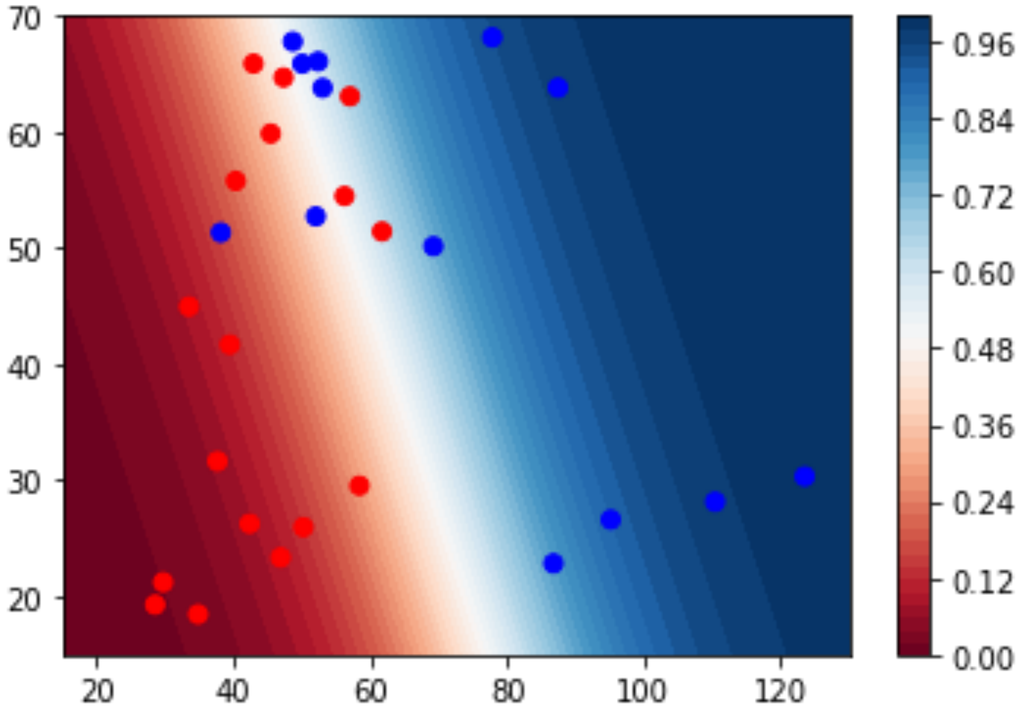
$$score(income, age) = \theta_0 + \theta_1 \times income + \theta_2 \times age$$

$$V_{model} = \sigma(score)$$

Nothing else to change:
Everything we have seen about cost, loss, gradient descent work the same

Visualizing the decision boundary

- If we represent the value outputted by the classifier for all age and all income, we can visualize the boundary decision

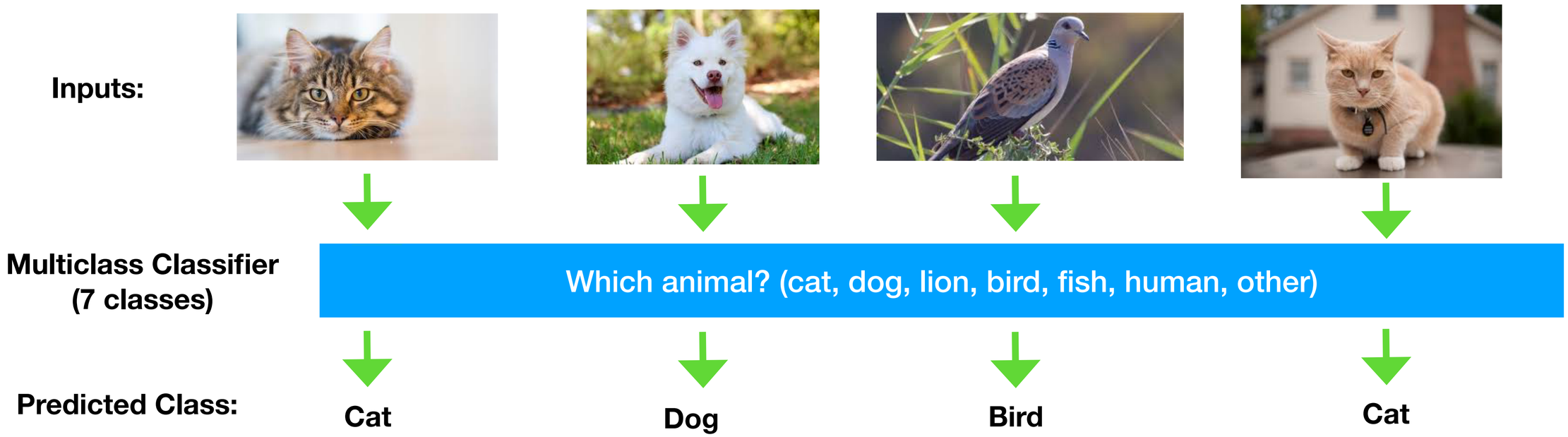


More complex models

- In fact, the score that we compute is like the linear models that we were using for linear regression.
- Therefore, we can apply the same tricks in term of feature expansion to learn a more complex model
- But beware overfitting

Multiclass Classification

- Remember that multi class classification is when we have more than 2 possible answers



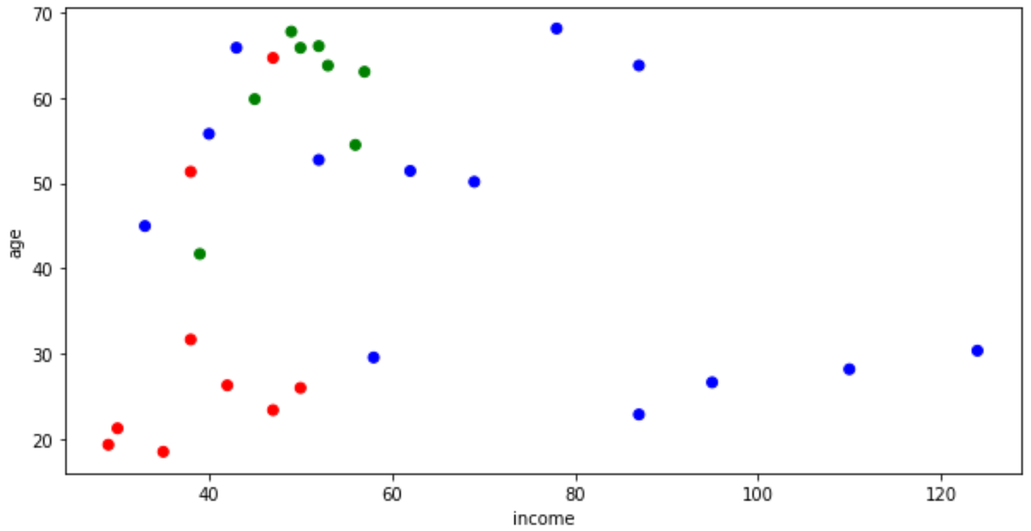
Multiclass Classification

- How do we do multi class classification?
- In practice, the idea is very similar
- Main difference: we are going to compute a ***score*** function for each of the classes

Multiclass Classification

- Let us now suppose that we have 3 parties: Left-Wing, Right-Wing and “Green”
- How are we going to predict the votes?

	income	age	vote
0	39.0	42.0	G
1	30.0	21.0	L
2	47.0	65.0	G
3	69.0	50.0	R
4	52.0	53.0	R
5	110.0	28.0	R
…	…	…	



Multiclass Classification

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2	47.0	65.0	G
3	69.0	50.0	R
4	52.0	53.0	R
5	110.0	28.0	R
…	…	…	

$$score_L(income, age) = \theta_0^L + \theta_1^L \times income + \theta_2^L \times age$$

$$score_R(income, age) = \theta_0^R + \theta_1^R \times income + \theta_2^R \times age$$

$$score_G(income, age) = \theta_0^G + \theta_1^G \times income + \theta_2^G \times age$$

We now have 3 x 3 = 9 parameters

$$V_{model} = softmax(score_L, score_R, score_G)$$

The softmax function

- The ***softmax*** function (sometimes called *multinomial logistic function*) is used for multiclass classification
- The input of a softmax function is a vector of N dimensions
- The output is a N-dimension vector that can represents probabilities for each class
- It is computed by taking the exponential of the components of a vector, then dividing by the sum of the exponentiated components

$$softmax(\overrightarrow{x})_i = \frac{exp(x_i)}{\sum_j exp(x_j)}$$

The softmax function

- Example: computing $softmax([1, -1, 0, 2])$

$$[1, -1, 0, 2] \xrightarrow{\text{1- Exponential}} [e^1, e^{-1}, e^0, e^2] \approx [2.71, 0.36, 1, 7.39]$$

2- Sum:

$$2.71 + 0.36 + 1 + 7.39 = 11.46$$

3- Divide by sum

$$\left[\frac{2.71}{11.46}, \frac{0.36}{11.46}, \frac{1}{11.46}, \frac{7.39}{11.46}\right] = [0.23, 0.03, 0.09, 0.65]$$

Softmax

The softmax function

- Example: computing $softmax([1, -1, 0, 2])$

$$[1, -1, 0, 2] \xrightarrow{\text{1- Exponential}} [e^1, e^{-1}, e^0, e^2] \approx [2.71, 0.36, 1, 7.39]$$

This step makes all values > 0

2- Sum:

$$2.71 + 0.36 + 1 + 7.39 = 11.46$$

3- Divide by sum

$$\left[\frac{2.71}{11.46}, \frac{0.36}{11.46}, \frac{1}{11.46}, \frac{7.39}{11.46}\right] = [0.23, 0.03, 0.09, 0.65]$$

This step makes all values sum to 1

Softmax

Positive values summing to 1: Can be seen as probabilities

Multiclass Classification

- Let us now suppose that we have 3 parties: Left-Wing, Right-Wing and “Green”
- How are we going to predict the votes?

	income	age	vote
0	39.0	42.0	G
1	30.0	21.0	L
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…	…	…	

We now have 3 x 3 = 9 parameters

$$score_L(income, age) = \theta_0^L + \theta_1^L \times income + \theta_2^L \times age$$

$$score_R(income, age) = \theta_0^R + \theta_1^R \times income + \theta_2^R \times age$$

$$score_G(income, age) = \theta_0^G + \theta_1^G \times income + \theta_2^G \times age$$

$$V_{model} = softmax(score_L, score_R, score_G)$$

This is now a vector of dimension 3 that contains probabilities of voting for each party

Example

$\theta_0^L = -3$ $\theta_1^L = 0.1$ $\theta_2^L = 0.2$

$\theta_0^R = -5$ $\theta_1^R = 0.3$ $\theta_2^R = 0.4$

$\theta_0^G = 0$ $\theta_1^G = 0.1$ $\theta_2^G = 0.1$

$$score_L(income, age) = \theta_0^L + \theta_1^L \times income + \theta_2^L \times age$$

$$score_R(income, age) = \theta_0^R + \theta_1^R \times income + \theta_2^R \times age$$

$$score_G(income, age) = \theta_0^G + \theta_1^G \times income + \theta_2^G \times age$$

$$V_{model} = softmax(score_L, score_R, score_G)$$

Somebody is 40 year old, with income of 50
Which party is predicted by this model?

Example

$\theta_0^L = -3$ $\theta_1^L = 0.1$ $\theta_2^L = 0.2$

$\theta_0^R = -5$ $\theta_1^R = 0.3$ $\theta_2^R = 0.4$

$\theta_0^G = 0$ $\theta_1^G = 0.1$ $\theta_2^G = 0.1$

$$score_L(income, age) = \theta_0^L + \theta_1^L \times income + \theta_2^L \times age$$

$$score_R(income, age) = \theta_0^R + \theta_1^R \times income + \theta_2^R \times age$$

$$score_G(income, age) = \theta_0^G + \theta_1^G \times income + \theta_2^G \times age$$

$$V_{model} = softmax(score_L, score_R, score_G)$$

Somebody is 40 year old, with income of 50
Which party is predicted by this model?

$$score_L(50,40) = -3 + 0.1 \times 50 + 0.2 \times 40 = 15$$

$$score_R(50,40) = -5 + 0.3 \times 50 + 0.4 \times 40 = 26$$

$$score_G(50,40) = 0 + 0.1 \times 50 + 0.1 \times 40 = 9$$

$$softmax([15,26,9]) = \frac{1}{e^{15} + e^{26} + e^{9}}[e^{15}, e^{26}, e^{9}] = [0.000199,0.9998,0.000001]$$

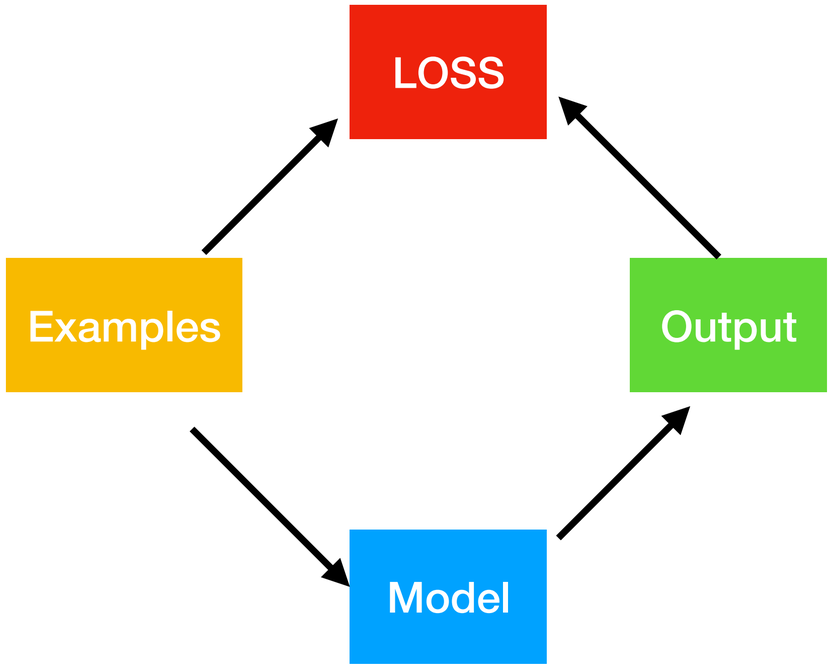
This model says he has 99.9% chances of voting for the right wing party

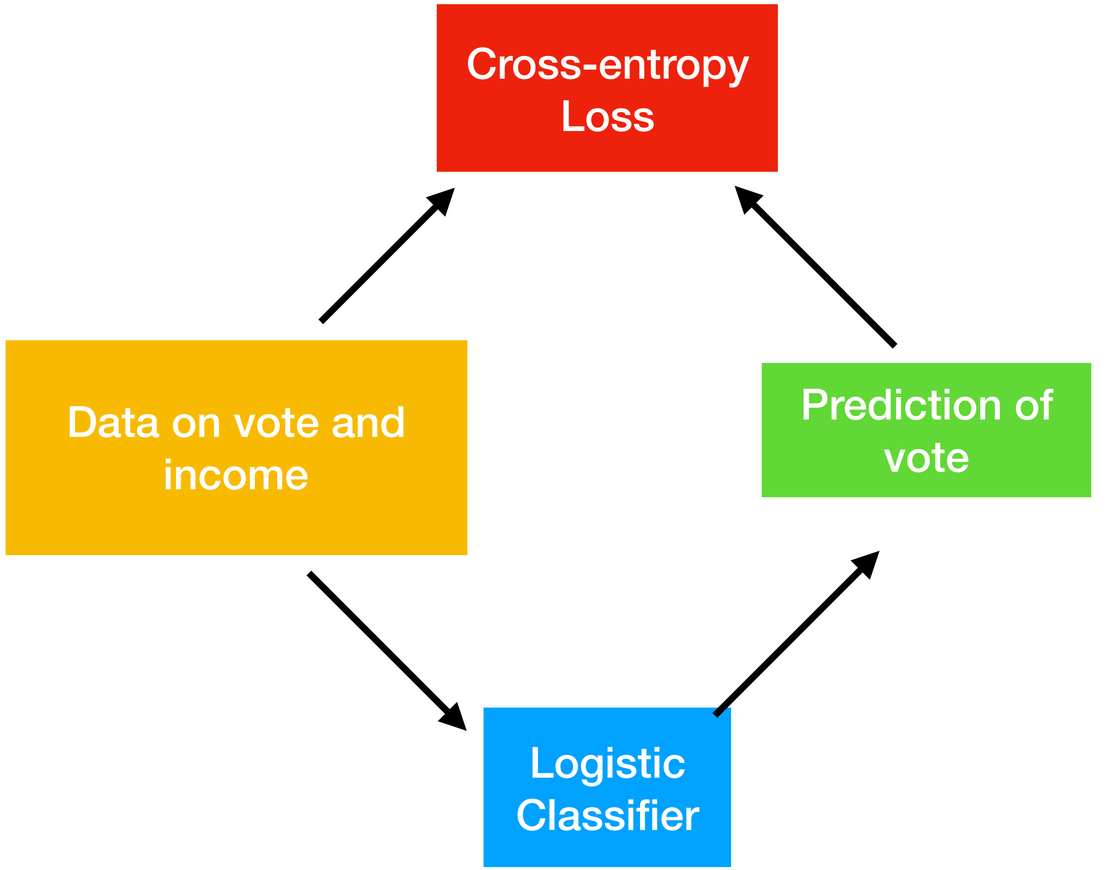
Multiclass Classification

- Finding the optimal parameters is once again very similar to the case of binary classification

- We will use a cross-entropy loss again

How to find the parameters θ?





We have some examples

We have a model with some parameters

We have a loss function that compute the difference between the example and the prediction of our model

We minimize the loss to obtain the best parameters for our model

Multiclass Classification

- Finding the optimal parameters is once again very similar to the case of binary classification
- We will use a cross-entropy loss again (adapted for multi-class case)

  - For each example:
    - If the correct class is **i**, then the ***cost*** of the prediction -log(**V[i]**)
  - Then we average the cost over each example

Next time

- Next time, we will start discussing the fancy things:
  - Neural Networks and Deep Learning
- We will have to discuss some mathematical topics as well (matrix and linear algebra)